IS THE CORRELATION BETWEEN FLUX TUBE EXPANSION FACTOR AND SOLAR WIND SPEED NEAR THE EARTH CAUSAL?

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ABSTRACT

The solar wind speed near the Earth has been predicted using an empirical relation between the flux tube expansion factor (FTE) and the solar wind speed observed near the Earth. However, the computation of FTE is based on potential field source surface (PFSS) model of the corona which has arbitrary dependence on parameters such as the number of spherical harmonics and the inverse mapping technique which do not include the effect of stream–stream interaction taking place between the source surface and 1 AU (Poduval and Zhao, 2004). Also, the source surface magnetic field obtained using the PFSS model varies from one point to the other contrary to the Ulysses observation of the heliospheric magnetic field, which is latitude independent. Therefore, the correlation between FTE and solar wind speed observed near the Earth, is likely to be coincidental rather than causal. If at all there exists a relation between FTE and solar wind, it must be the wind near the Sun rather than near the Earth. The Horizontal Current Current–Sheet Source Surface (HCCSSS) model developed by Zhao and Hoeksema (1995) yields a uniform magnetic field and has been shown to reproduce the radial variation of non–radial mid–latitude helmet streamers between 2.5 and 30 $R_\odot$. We present the results of an investigation of the validity of the inverse correlation between the solar wind speed and FTE using our HCCSSS model.

Key words: solar wind; PFSS model; Wang and Sheeley model; flux expansion factor.

1. INTRODUCTION

An inverse correlation between the solar wind speed (SWS) observed at 1 AU and the flux tube expansion factor very close to the Sun ($r < R_{ss} = 2.5R_\odot$) has been established by Wang and Sheeley (Wang and Sheeley, 1990; Wang and Sheeley, 1992; Wang and Sheeley, 1994; Wang, 1995; Wang et al., 1997). This inverse relation has been made use of in the prediction of SWS at 1 AU using potential field source surface (PFSS) model of the corona (Arge and Pizzo, 2000). The predicted SWS does not agree with the observed speed near the Earth always and the discrepancies are very significant as pointed out by several authors (Bala, 2000, for instance). Using the observed SWS near the Earth and the potential field source surface (PFSS) model of the corona, Poduval and Zhao (2004) have shown that the inverse mapping of the solar wind from the point of observation in the heliosphere to the source surface can influence the determination of the source surface location of the solar wind and thereby the computation of flux tube expansion factor (FTE) which in turn can lead to a wrong prediction of the solar wind speed. Also, the number of multipole components used in the spherical harmonic expansion of the observed solar magnetic field which goes into the PFSS model as input can influence the determination of the footpoints of coronal sources of solar wind and in turn, the FTE.

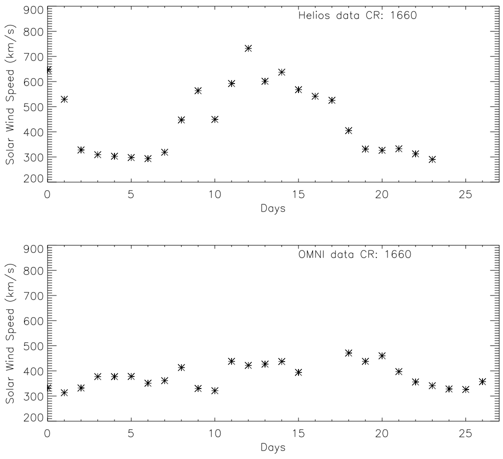

Figure 1. The solar wind profiles at Helios I (top panel) and near the Earth (bottom panel) for CR 1660.

We hypothesize that if there exists a relation between SWS and FTE, it must be the wind near the Sun rather than that near the Earth. This is because, due to the interaction between fast and slow solar wind streams during their propagation from the corona to 1 AU and beyond, the solar wind profile gets altered. In fact, the Helios data presented a solar wind profile near the Sun that has only two distinct components, the high speed wind separated sharply by a low speed wind (Schwenn, 1990).

In the present work, we made an attempt to see the correlation beteween FTE and SWS observed by HELIOS I and II spacecraft. For a comparison, we used the near–Earth solar wind data observed during the same period. We used both the PFSS model as well as the Horizontal Current Current Sheet Source Surface (CSSS) model of the corona to compute FTE.

1.1 Flux Tube Exapnsion Factor

The flux tube expansion factor FTE is defined as:

$$FTE = \left(\frac{R_\odot}{R_{ss}}\right)^2 \frac{B_r(\theta_\odot, \phi_\odot)}{B_r(\theta_{ss}, \phi_{ss})} \qquad (1)$$

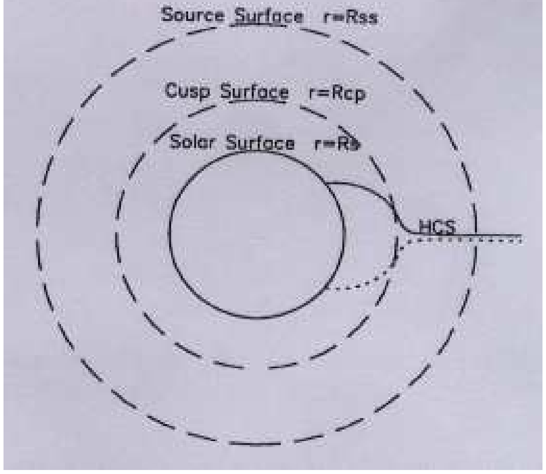

Figure 2. The geometry of the CSSS model.

where, $B_r(\theta_{ss}, \phi_{ss})$ denotes the magnetic field strength at location (θ_{ss} and ϕ_{ss}) on the source surface and $B_r(\theta_\odot, \phi_\odot)$ is the field strength at the photospheric footpoint of the flux tube traversing (θ_{ss} and ϕ_{ss}). $R_\odot$ and R_{ss} are the photospheric and source surface radii, respectively. In order to correlate with FTE, the solar wind observed in the heliosphere is traced back to the source surface using the following set of equations:

$$\begin{aligned} \phi_{ss} &= \phi_E + \frac{\omega R_E}{V} \\ \theta_{ss} &= \theta_E \end{aligned} \tag{2}$$

where, θ_{ss}, ϕ_{ss} and θ_E, ϕ_E are the heliographic latitudes and Carrington longitudes of a point at the source surface and at distance R_E from the Sun, respectively; ω, the angular speed of solar rotation and V, the solar wind speed.

Usually, an approximate value for V is used, for all the observed solar wind speed. We call it `constant speed approximation`. That is, a constant value of 4, 4.5 or 5 days, the average transit time of solar wind from the Sun to Earth, will be assigned to V. The influence of `constant speed approximation` on the determination of the coronal source of solar wind and thereby on the computed FTE has been discussed in detail by Poduval and Zhao (2004). In that paper, we concluded that it is reasonable to use the observed daily averaged values of solar wind in Eq. 2.

2. CORONAL MODELS

2.1 Potential Field Source Surface Model

In the lower corona, the total magnetic energy density is much larger than the energy density of the plasma. Therefore, the plasma motion as well as the structure of the corona are determined by the magnetic field. Above a certain height, the plasma is assumed to overcome the magnetic field and therefore, the field lines are constrained tp be radial by the outward propagating solar wind. The field between the solar surface and the source surface can be represented by a scalar potential if the currents carried by plasma are negligible. Then, with the radial field at the source surface and the observed photospheric field as boundary conditions the coronal magnetic field can be computed. Such a model of the corona, called the Potential Field Source Surface Model (PFSS or SS), first put forth by Schatten et al., (1969) and Altschuler and Newkirk (1969), independently, has been useful in predicting the observed structures, though a certain level of discrepancy exist, especially in the finer details. The biggest drawback of SS model is that it is highly sensitive to the rapid evolution of the photospheric field. The model is also sensitive to the resolution of the photospheric magnetic field as well as some other parameters in the model (for details, see Hoeksema, 1984).

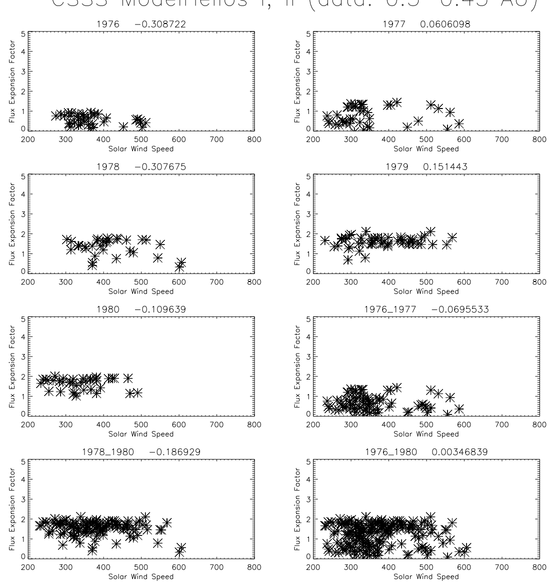

Figure 3. Scatter plot of log_{10}(FTE) and SWS using CSSS model and Helios I and II data, within 0.29–4.0 AU.

The SS model determines the coronal magnetic field from the photosheric magnetic field by solving the LaPlace's equation in terms of the spherical harmonic functions. The number of terms (represented by N_{max} included in the spherical harmonic expansion can alter the accuracy of the final results, especially regions near the photosphere. The larger the value of N_{max} the better the accuracy. However, the spatial resolution of the input magnetic field data determines an upper limit to N_{max}. For example, Wilcox Solar Observatory (WSO) data, which has a spatial resolution of 5° X 5°, limits N_{max} to 23.

The SS model has been used to calculate FTE, given in Eq. 1, which is later used to predict SWS at 1 AU using an empirical relation obtained between FTE and SWS (Arge and Pizzo, 2000; Wang and Sheeley, 1990). Though the predicted SWS agrees with the observed SWS at 1 AU at time, the discrepancies are very significant (e.g. Bala, 2000). Poduval and Zhao (2004) have shown that the number of multipole componets (N_{max}) used in the

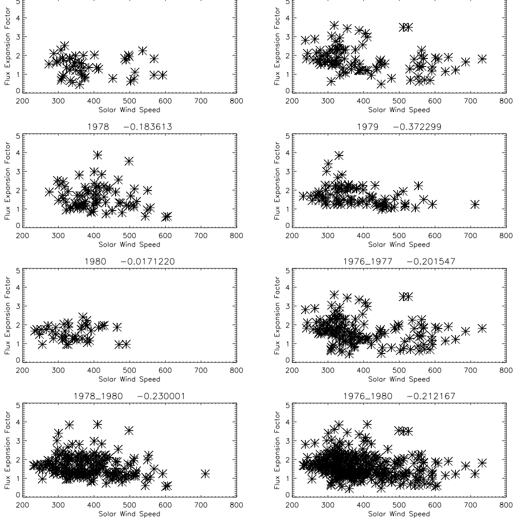

Figure 4. Same as Figure 3 but for SS model.

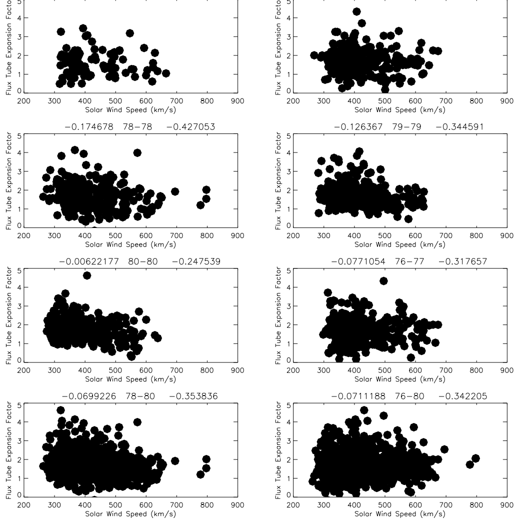

Figure 5. Scatter plot of log_{10}(FTE) and SWS using SS model and solar wind data near the Earth.

spherical harmonic expansion of the solar magnetic field in the SS model can significantly influence the computation of FTE which in turn alters the predicted SWS.

2.2 Horizontal Current Current Sheet Source Surface Model

The SS model, described above, approximates the coronal field to a potential field. Also, the model takes into account only the effect of volume currents flowing beyond the source surface and neglects currents (a) between closed and open magnetic fields and (b) in the heliospheric current sheet (HCS). This causes discrepancies between model and observation. Another limitation of SS model is that the source surface is located at 2.5 $R_{\odot}$, a distance much lower than the Alfvén critical point, where all field lines become radial. The Horizontal Current Current Sheet Source Surface Model, HCCSSS or CSSS, developed by Zhao and Hoeksema (1995), takes all these aspects into account.

In the CSSS model, the corona is treated as a magnetostatic atmosphere with horizontal electric currents and the solutions obtained by Bogdan and Low (1986) has been made use of. Here, the coronal atmosphere is devided into three regions, (a) bounded by the photosphere and a cusp surface, located at the cusp point of the coronal streamers, (b) between the cusp surface and a source surface, within which all magnetic field is open but need not be radial and (c) beyond the source surface all the magnetic field is radial. Figure 2 shows the geometry of the CSSS model. The cusp surface models the effects of streamer current sheets and the source surface models the effects of volume currents beyond the source surface. The model has three free parameters, one representing the height distribution of the horizontal current, a, the second representing the heliocentric distance of the cusp surface, R_{cp} and the third, the heliocentric distance R_{ss} of the source surface. For the present computation we have chosen the following parameters: $a = 0.2$, $R_{cp} = 2.0$, $R_{ss} = 15.0$ (Zhao et al., 2002). Note that the source surface is placed near the Alfvén critical point, where all the magnetic field lines are radial. This is a great improvement over the SS model, since, in SS model, the source surface is place at 2.5 $R_{\odot}$, a distance much lower than the Alfvén critical point, where the field lines are forced to be radial.

Using the CSSS model, Zhao et al., (2002) were able to reproduce the radial variation of helmet streamers during the ascending phase of solar cycle. Also, the CSSS model yields uniform magnetic field on the source surface, which is consistent with the Ulysses observation (Zhao et al., 2001).

2.3 Solar Wind Data

In order to see the relationship between FTE and the solar wind near the Sun, we used the HELIOS I and II data during 1976-1980. The data covers a heliocentric distance of 0.29–0.99 AU. We selected the data within 0.4 AU, a region where the interaction between the fast and slow wind is negligibly small. For a comparison, we also used the near–Earth data from the OMNI archive for the same period.

The spacecraft latitude has been taken as the latitude on the source surface, assuming radial propagation of solar wind, while using the HELIOS data, whereas the b0 angle at the observed solar wind location for the OMNI data. Inorder to get the Carrington Longitude on the source surface, the solar wind at HELIOS/Earth has been traced back assuming radial flow along the Archemedian spiral (Eq. 2). We used the observed daily values for this purpose. The separation between the spacecraft and Earth is also taken into account while computing the Carrington longitude.

3. SCATTER PLOTS

Figures 3–5 depict the scatterplots of SWS *Vs* log_{10}(FTE), plotted different phases of the solar cycle. The left hand panels represent the periods, 1976, 1978, 1980 and 1978-1990 while those on the right show 1977, 1979, 1976-1977 and 1976-1980. In Figure 3, the FTE is calculated using CSSS Model and the near–Sun solar wind data, Helios I and II, within a heliocentric distance range of 0.29–0.4 AU. Using the same set of data we computed FTE using the SS model and is depicted in Figure 4. For a comparison, we made a scatterplot using the near–Earth solar wind data for the same period and FTE computed using the SS model. Though there is a general trend of an inverse correlation in Figure 5, there is a large scatter around that. Moreover, note that this general trend is not so clear in Figure 3 and 4.

The correlation coefficient obtained for all the cases are summarized in Table 1. The first two columns represent the correlation coefficient using the near–Sun solar wind data from Helios and FTE computed using CSSS and SS models, respectively, while the last column shows correlation coefficient using near–Earth solar wind data and SS model. Note that the correlation is very low and inconsistent for all cases. In the case of near–Sun data, Helios, for those data (year) where CSSS model yielded a higher (lower) correlation, the SS model shows lower (higher) correlation, in general. Again, while the correlation (CSSS model) is higher in 1976 and 1978 (-0.305 and -0.306, respectively) all the remaining periods showed a much smaller correlation. In the case of SS model 1977 and 1979 shows higher correlation (-0.272 and -0.372, respectively). When all the data is considered the CSSS model yielded poor correlation (0.004) whereas SS model gave slightly higher correlation (-0.213). Splitting the data according to minimum (1976-77) and ascending (1978-80) phases of solar cycle did not improve the scenario. The former yielded a correlation coefficient -0.071 (CSSS) and -0.202 (SS model) whereas the latter gave -0.186 and -0.230 for CSSS and SS models respectively. Note that correlation coefficient is much higher (but still $<$ 0.5) for the near–Earth solar wind data, for all periods. This is the most interesting point in this study and will be discussed in the next section.

4. DISCUSSION AND CONCLUSION

The inverse relation between SWS and FTE computed at 2.5 $R_\odot$ using SS model, established by Wang and Sheeley (1990) has been made use of in the prediction of solar wind at 1 AU (Arge and Pizzo, 2000). The predicted solar wind speed does not always agree with the observed one and the discrepancies are significant. Poduval and Zhao (2004) have found that the correlation between SWS and FTE is not very significant nor is consistent always. The coronal magnetic field computed using SS model is highly structured and exhibits a profile similar to that of SWS at 1 AU, contrary to the Ulysses observation of a rather uniform heliospheric magnetic field. This similarity appears to be coincidental rather than causal since the solar wind profile at 1 AU results from the interaction between fast and slow winds during their outward propagation from the corona and is different from the profile near the Sun where the interaction is negligible (See Figure 1 and Schwenn, 1990). Based on the two types of stream–stream interaction, we speculated that the apparent correlation between SWS and FTE could be coincidental rather than causal (Poduval and Zhao, 2004). This speculation is partially supported by the fact that the coronal magnetic field computed using CSSS model is rather uniform (Zhao et al., 2001), consistent with Ulysses observation. We hypothesised that if there exists a physical relationship between FTE and SWS, then FTE must correlate with the SWS near the Sun, where the stream–stream interaction is negligible (Poduval and Zhao, 2004). To check this hypothesis, we used the Helios I and II data within 0.4 AU to obtain the correlation between FTE and SWS.

Table 1. Correlation coefficientbetween log_{10}FTE and SWS using Helios and OMNI data.

Period of Study	Helios I and II Data		OMNI Data
	CSSS Model	SS Model	SS Model
1976	-0.305	-0.074	-0.398
1977	0.058	-0.272	-0.281
1978	-0.306	-0.184	-0.427
1979	0.151	-0.372	-0.345
1980	-0.109	-0.017	-0.248
1976-77	-0.071	-0.202	-0.318
1978-80	-0.186	-0.230	-0.354
1976-80	0.004	-0.213	-0.342

From the present analysis, we find that the correlation between FTE and the near–Sun SWS is also weak and insignificant. Neither SS nor CSSS model yielded a significant correlation during the period of study. This fact strongly suggests that the correlation between FTE and SWS near the Sun is not causal. Moreover, it is to be noted that the correlation between SWS near the Earth and FTE using SS model is higher than the correlation between SWS near the Sun and FTE. This further supports our speculation that the greater correlation of FTE with near–Earth solar wind is due to the fact that solar wind profile at 1 AU is modified by Type I stream–stream interaction, which coincidentally matches with the coronal magnetic field profile.

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